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STABILITY OF SEVERE SLUGGING

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Abstract—For a constant flow rate of liquid and gas in a pipe one expects the conditions along the pipe to be of a steady state nature. However, for a pipe in a hilly terrain or in an offshore pipeline-riser system, a steady state operation is often not possible, and conditions of severe or terrain slugging develop. This causes the system to operate in an undesired cyclic fashion in which alternate long liquid slugs are followed by the production of high gas flow rate. The present work deals with the condition under which steady state operation is possible. It shows theoretically that it is possible to stablize the flow by increasing the back pressure of the separator or by employing a controlled choking at the pipe exit.

INTRODUCTION

Steady state operation of two-phase flow in pipes usually means that the flow rate of liquid and gas are constant. As a result, the conditions at any point in the pipe remain constant; namely, the flow pattern, average void fraction, average pressure drop and average local flow rates do not vary with time.

The term average is used here because two-phase flow is seldom a truly steady state flow and averaging values are used over a time period characteristic of the flow pattern. A typical example is the slug flow pattern, for which average values are taken during one or a few slug passages.

However, under certain situations a steady state operation is not possible. For example, when a subsea line with downwards inclination ends with a vertical riser to a platform, or when a pipe is laid in a hilly terrain, under certain conditions the lower section of the pipe accumulates liquid and blocks the gas passage. The gas upstream is compressed until it overcomes the gravitational head of the liquid, thereby creating a long liquid slug that is pushed in front of the expanding gas upstream. Under such conditions a cyclic operation is obtained, termed severe or terrain slugging. Severe slugging is considered to be an unstable flow regime in the sense that it is associated with large and abrupt fluctuations in the pipe pressure and in the gas and liquid flow rates at the outlet.

The process of severe slugging formation can be described as taking place according to the following steps.

The first step is the slug formation (figure 1). In this step liquid entering the pipeline accumulates at the bottom of the riser, blocking the gas passage and causing the gas to compress. When the liquid height in the riser, z reaches the top of the riser, $z = h$, the second step of slug movement into the separator starts (figure 2). After the gas that is blocked in the pipeline reaches the bottom of the riser, the liquid slug continues to flow into the separator with a rather fast velocity, termed blowout (figure 3). In the last step, figure 4, the remaining liquid in the riser falls back to the bottom of the riser and the process of slug formation starts again.

The severe slugging pattern is typical of relatively low liquid and gas flow rates. It requires that the flow pattern in the pipeline be stratified. In addition, it requires that the liquid reaches the top of the riser pipe before the gas reaches the bottom of the riser during slug formation. The latter condition can be calculated using the Schmidt *et al.* model (1980). A simplified version of the Schmidt model is used here (appendix A) to determine the flow rate of liquid and gas at which severe slugging will not occur.

Severe slugging is an undesired phenomenon. One of the methods of alleviating severe slugging is by increasing the separator back pressure (Yocum 1973). Choking the flow (Schmidt *et al.* 1979b, 1980) was also found to alleviate severe slugging with minimal increase in the pipeline pressure (for the same flow rates of liquid and gas). Once severe

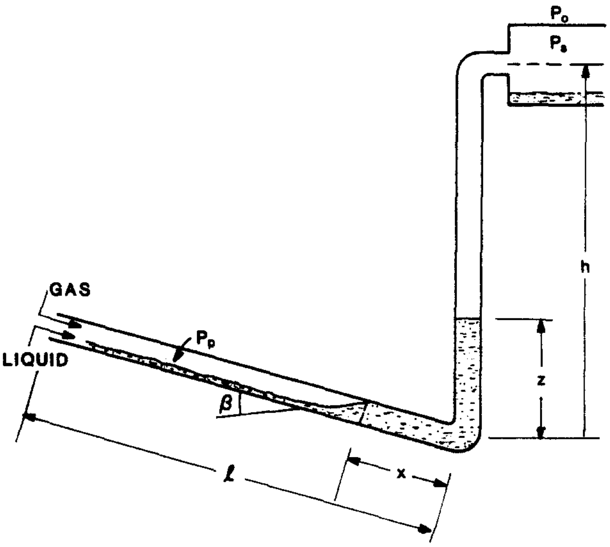

Figure 1. Slug formation.

slugging was eliminated, a steady state operation was achieved as shown in figure 5. In this steady state operation the pipeline is in stratified flow while the riser is in bubble or slug flow. The pressure of the pipeline remains constant and the liquid does not penetrate upstream into the pipeline to form the long liquid slug.

In spite of the progress achieved in eliminating severe slugging, it seems that this process is not well understood and the conditions under which severe slugging can be transformed into steady state flow are still not clear. The statment that "the process in which severe slugging has been eliminated successfully has been repeated often enough to prove the value of choking as probably the most practical method of eliminating slugging" (Schmidt *et al.* 1980), reveals the need for a better understanding of this process.

In this work we examine the conditions under which severe slugging will take place and find under what conditions and how severe slugging could be eliminated and transformed into steady state operation. Furthermore, the stability of steady state operation is analysed and the conditions under which steady state operation will take place are established.

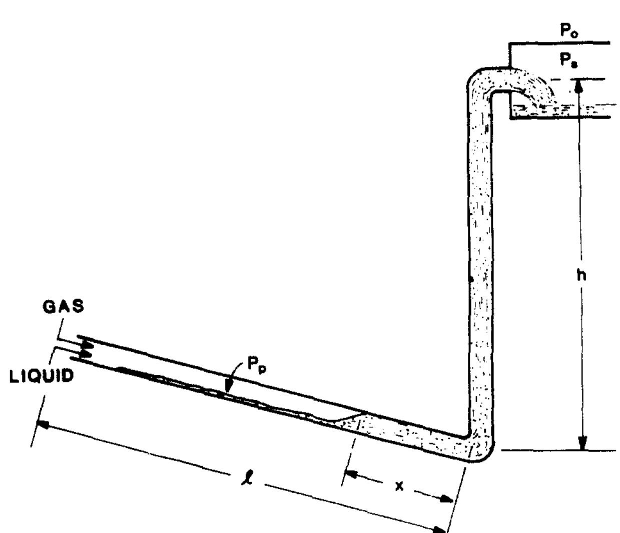

Figure 2. Slug movement into the separator.

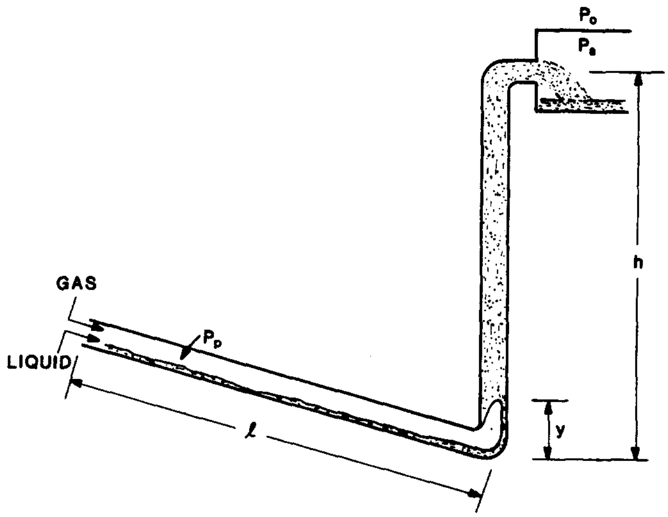

Figure 3. Blowout.

ANALYSIS

Severe slugging occurs due to the compressibility of the gas. The gas compressibility manifests itself in the blowout step of the severe slugging cycle (figure 3). In this step the liquid column height is reduced and an unstable situation can be reached where the pressure in the pipeline, p_p, will exceed the back pressure provided by the separator and the liquid column ($h-y$). If the system is not stable the liquid will be blown-out by the gas, thereby causing the severe slugging cycle to take place.

This situation can be analysed as follows: Assume that the cycle of severe slugging reaches the point at which the slug tail has just entered the riser and the riser is now liquid full. Assume a small disturbance y that may carry the liquid somewhat higher (see figure 3, where y can also be considered the disturbed level) and that the disturbance is fast enough so that the slow flow rate of liquid and gas is ignored while y changes.

The net force (per unit area) acting on the liquid in the riser is

$$\Delta F = [(P_s+\rho_L gh)\frac{\alpha l}{\alpha l+\alpha' y}] - [P_s+\rho_L g(h-y)] \quad . \qquad [1]$$

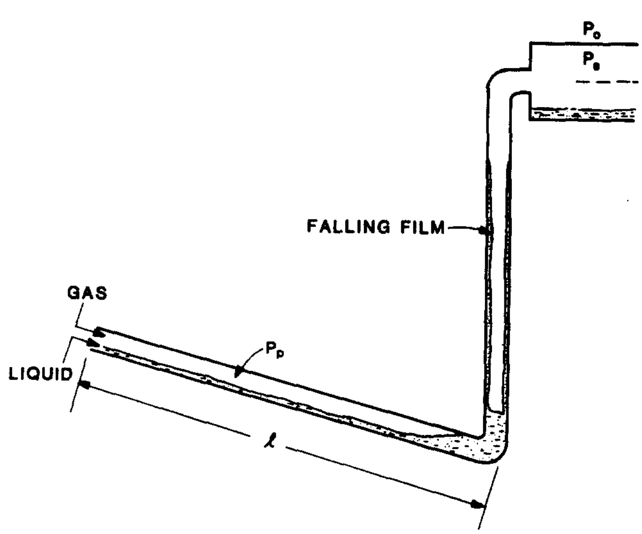

Figure 4. Liquid fallback.

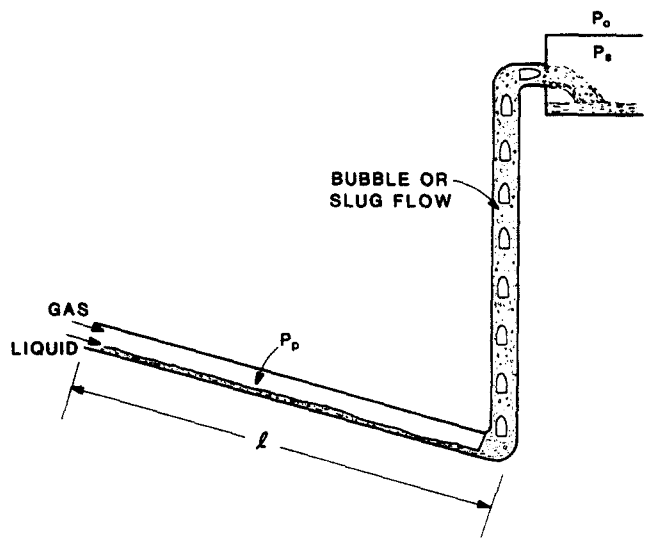

Figure 5. Steady state operation.

The first term on the rhs in the square parenthesis is the pipeline pressure driving force. The pressure varies with y as a result of the expansion of the gas in the pipeline. The second term corresponds to the back pressure force applied by the separator pressure and the liquid column of density ρ_L and height $(h-y)$. Note that for $y=0$ the system is in equilibrium and $\Delta F=0$. l and h are the pipeline and riser lengths, respectively. P_s is the pressure in the separator. α is the gas holdup in the line which is in stratified flow. α' is the gas holdup in the gas cap penetrating the liquid column. α can be calculated on the basis of a stratified flow model described in appendix B. α' is calculated on the basis of the slug flow model described in appendix C. α and α' have values typically ranging from 0.8 to 1.0. Their exact values only slightly effect the results. In this analysis, shear stresses are neglected due to the low rates typical of severe slugging operation. Also, the gas is assumed to expand isothermally following the "ideal gas" law.

The liquid column will be blown out of the pipe if ΔF increases with y, which is a necessary condition for severe slugging flow. Thus the condition for stability, namely the condition under which severe slugging is not possible is

$$\frac{\partial(\Delta F)}{\partial y} < 0 \text{ at } y=0 \quad . \qquad [2]$$

This leads to the criterion for stability

$$\frac{P_s}{P_0} > \frac{(\alpha/\alpha')l - h}{P_0/\rho_L g} \quad , \qquad [3]$$

where P_0 is the atmospheric pressure.

This is a very simple result stating that when the separator pressure increases to the level that satisfies [3], severe slugging will be eliminated and steady state condition will be reached. It is also interesting to observe that the system becomes less stable for increasing pipeline length and more stable for increasing riser length.

Stability of steady state operation

The stability of the steady state operation shown in figure 5 could be analysed the same way, except with the liquid density replaced by the average column density. Designating the liquid holdup in the riser as ϕ, the average density (neglecting the gas density) is $\phi\rho_L$ and [3] takes the form

$$\frac{p_s}{P_0} > \frac{\phi\,((a/a')l - h)}{P_0/\rho_L g} \quad . \qquad [4]$$

Since ϕ is less than unity, this result indicates that steady state operation is more stable than step 3 in the severe slugging cycle. This also suggests that lower separator pressure can be used once steady state operation is reached.

The analysis of the stability of a steady state operation requires, however, the knowledge of the average liquid holdup in the pipe, ϕ. This can be calculated on the basis of steady state models or correlations which yield the liquid holdup as a function of the operating conditions.

Steady state operation

In steady state operation, the flow in the riser pipe will take the form of either bubble of slug flow. The average liquid holdup, ϕ, depends on the liquid superficial velocity, gas mass flow rate (or superficial velocity under atmospheric conditions) and the separator pressure. It is convenient, however, to express this functional relationship in the form of (as in [4])

$$\frac{P_s}{P_0} = f(\phi,\ U_{LS},\ U_{GS0}) \quad , \qquad [5]$$

where U_{LS} is the superficial velocity of the liquid (assumed incompressible) and U_{GS0} is the superficial velocity of the gas under atmospheric condition P_0.

For this we need models for bubble flow and slug flow that will result in the relation given in [5]. This relationship is described below for bubble flow. The more complex relation for slug flow is given in appendix C.

For bubble flow the liquid and gas velocities are

$$U_G = \frac{U_{GS}}{1-\phi} \quad , \qquad [6]$$

$$U_L = \frac{U_{LS}}{\phi} \quad , \qquad [7]$$

where ϕ is the liquid holdup. The superficial gas velocity depends on the separator pressure, $U_{GS} = U_{GS0}P_0/P_s$. Assuming that the slip velocity $U_0 = U_G - U_L \simeq$ constant, the following relation is obtained,

$$\frac{P_s}{P_0} = \frac{U_{GS0}}{(U_0 + \frac{U_{LS}}{\phi})\,(1-\phi)} \quad , \qquad [8]$$

where U_0 can be calculated by (Harmathy 1960)

$$U_0 = 1.53\left[\frac{g(\rho_L - \rho_G)\sigma}{\rho_L^2}\right]^{1/4} \quad . \qquad [9]$$

Note that for simplicity the gas density in the riser is approximated by the gas density at the separator pressure.

The analysis of the possible steady states and their stability can be demonstrated with an example for an air–water system with superficial liquid velocity of $U_{LS}=0.1$ m/s and superficial gas velocity (at atmospheric conditions) U_{GS0} in the range of 0.05 to 0.2 m/s.

P_s/P_0 for steady state operation as a function of ϕ ([8]) is plotted in figure 6 for $U_{GS0}=0.05$, 0.1 and 0.2 m/s. The bubble flow pattern changes to slug flow at about liquid holdup $\phi \simeq 0.7$ (Taitel *et al.* 1980). Therefore the curves in figure 6 represent the result of

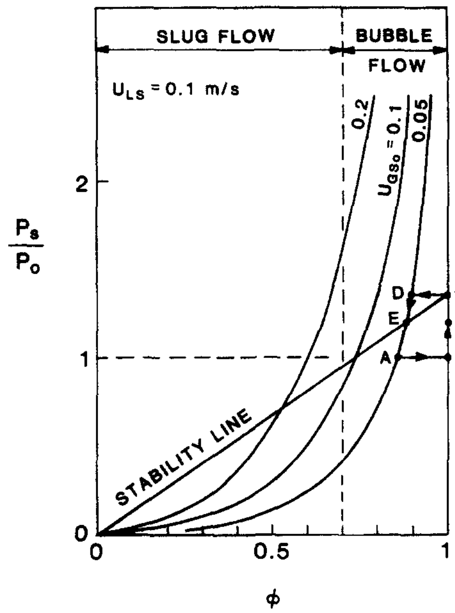

Figure 6. Stability analysis, water–air, $D=5$ cm diam., $l=30$ m, $h=15$ m, $\beta=2°$.

the bubble flow pattern from $\phi=1$ to 0.7 while for $\phi<0.7$ the model of slug flow is used (appendix C).

If we now plot the straight line of P_s/P_0 vs ϕ as given in [4], which represents the stability criteria for steady state operation, we can learn when the flow is stable. The straight line in figure 6, the stability line, represents the result of [4] for a riser length of 15 m and pipeline length of 30 m with 2° downwards inclination (a system similar to the one used by Schmidt *et al.* 1980). The void fraction in the pipeline α, calculated using Taitel & Dukler (1976) (see appendix B), was found to yield $\alpha=0.87$. α' can be calculated on the basis of the slug flow model since the Taylor bubble that penetrates the column is the same as Taylor bubbles in normal slug flow. To use the slug flow model (appendix C) for this purpose we need as input the flow rates of the liquid and gas, which are unknown for the disturbed variable. However, using the slug flow model for U_{LS} and U_{GS}, both ranging from 0.01 to 10 m/s, shows that the α' obtained is not sensitive to the flow rates and the result for this particular system (for all flow rates) was that practically $\alpha'=\text{constant}=0.89$. Note also that the exact value of α' is not important anyway.

Steady state operation above the stability line is stable whereas below it, it is unstable.

Since the separator pressure will always exceed the atmospheric pressure ($P_s/P_0>1$), the system will be stable for $U_{GS0}=0.2$ and 0.1. At atmospheric pressure for $U_{GS0}=0.2$ the riser will be under slug flow ($\phi<0.7$) whereas for $U_{GS0}=0.1$ m/s it will operate in bubble flow.

For $U_{GS0}=0.05$ m/s the system is unstable at atmospheric pressure. In order for the system to be stable in steady state operation the separator pressure should exceed 1.2 atmospheres.

We can now use this figure to analyse the system and examine its possible modes of operation. Suppose that $U_{GS0}=0.05$ and the separator pressure is atmospheric (point A in figure 6). The steady state system will be unstable and transition to severe slugging will take place. In severe slugging the liquid holdup is unity and we can consider B the point that represents the stability of this mode of operation. In order to eliminate severe slugging one can elevate the separator pressure to point C. The high pressure will stabilize the flow and a new steady state operation will develop (point D). As seen, however, point D is more stable than C and it is possible now to decrease the separator pressure to $\simeq 1.2$ atmospheres (point E) and the system will stay stable. It should be emphasized, however, that at 1.2 atmospheres the system can operate both under steady operation (point E) and severe slugging operation (F). Once the system is in the severe slugging pattern one has to increase temporarily the separator pressure to return to the steady mode.

Steady unstable operation

An unstable system can still operate around the equilibrium steady state provided a feedback control system is used to stabilize the system. Schmidt *et al.* (1980) found experimentally that they could stabilize the flow by choking the flow at the top of the riser before entering the separator (see figure 7).

Following this analysis it is clear that if choking can be used to increase the back pressure P_S proportionally to the disturbance movement y, a controlled stable system can result. Using a control system to provide

$$P_b - P_s = Ky \qquad [10]$$

the net force that acts on the column is now (see [1])

$$\Delta F = [(P_s + \phi\rho_L gh)\frac{\alpha l}{\alpha l + \alpha' y}] - [(P_s - Ky) + \phi\rho_L g(h - y)] \quad . \qquad [11]$$

The condition for stability given by [2] yields

$$\frac{P_s}{P_0} > \frac{\frac{\alpha}{\alpha'} l (\phi - \frac{K}{\rho_L g}) - \phi h}{P_0/\rho_L g} \quad . \qquad [12]$$

Equation [12] can be used to determine the desired combination of separator pressure and the stability coefficient K to ensure steady state operation.

The control system should be designed in such a way that the choking valve will be adjusted to provide an increase of the back pressure according to [10]. Such a control system needs an input of the value of the disturbance y. This could be done in various ways, either measuring y directly (using void fraction detectors) or by correlating the pressure difference between the bottom of the riser and some location higher than the y position.

It is interesting to observe that, to a good approximation, little movement of the choking valve is needed for such a control system. This makes it possible to set the choking valve in a precalculated constant value.

The control valve function approximately follows the relation

$$P_b - P_s = CU_M^2 \qquad [13]$$

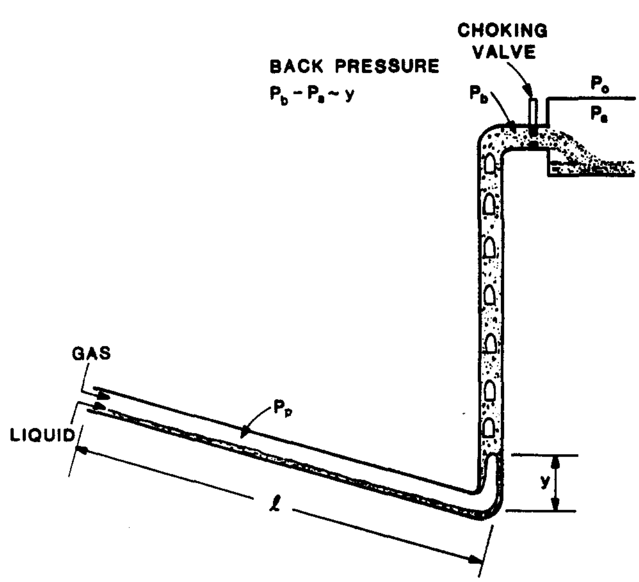

Figure 7. Stabilizing unstable steady state operation.

where U_M is the mixture velocity. Since $P_b - P_s = Ky$, the choking value setting C is given by

$$C = \frac{Ky}{U_M^2} \quad . \qquad [14]$$

Using quasi-steady-state approximation, ΔF as given by [11], causes the mixture to flow with the velocity U_M given by

$$\Delta F = \frac{4}{D} f \frac{\rho_M U_M^2}{2} (h - y) \quad . \qquad [15]$$

Using [11], ΔF for small y yields

$$\Delta F = [\phi \rho_L g - K - \frac{\alpha'}{\alpha l}(P_s + \phi \rho_L g h)]\, y \quad . \qquad [16]$$

Combining [15] and [16] to calculate U_M and substituting it in [14] yields (for small y)

$$C \simeq \frac{2 f \rho_M h K / D}{\phi\, \rho_L g - K \frac{\alpha'}{\alpha l}(P_s + \phi \rho_L g h)} \quad . \qquad [17]$$

Equation [17] demonstrates that, to a good approximation, C can be set at a constant value and the system will remain stable. This can provide the explanation for the success of choking to stabilize steady state flow as reported by Schmidt *et al.* (1979b, 1980).

COMPARISON WITH EXPERIMENTAL RESULTS

Very few experimental results are reported in the literature that can be used to compare with the present analysis. Schmidt (1977) reported the most comprehensive study on the severe slugging phenomenon which was also detailed by Bendiksen *et al.* (1982). In a systematic study Schmidt mapped the flow patterns observed in a riser pipeline system that is similar to the one used here. The experimental system consisted of 51-mm-diam. Lexon pipe with a 30-m-long inclined pipeline and 15-m-long vertical riser. Air and kerosene were used as the fluids, and the separator pressure was kept approximately at atmospheric pressure. Unfortunately the separator was placed on the ground and not at the top of the riser. The fluids entered the separator via a 15-m-long downcomer followed by a horizontal lead. As a result, the pressure at the top of the riser was not adequately controlled and it varied with the flow rates and the holdup in the downcomer (siphon effect) in the range of atmospheric pressure to about one-tenth of it. This is not quite consistent with the model that assumes a constant separator pressure at the top of the riser.

Although it is not possible to study the accurate parametric trend of this theory some comparison is still feasible. This fact, incidentally, shows the importance of developing a theory prior to experimentation in which case planning of the experiments could be more effective.

Figure 9 shows the experimental flow patterns observed in the riser as a function of the flow rates with a 5° pipeline inclination. Schmidt observed the following patterns: Severe slugging type I, severe slugging type II, transition to severe slugging, bubble flow, normal slug flow and annular flow. Of all of these flow patterns only the one designated as severe slugging I is the "true" severe slugging as per the definition that the slug length should exceed the riser length in severe slugging flow.

The broken line in figure 9 is Schmidt experimental boundary that demarcates the region that was observed experimentally as severe slugging I. The solid line a is a plot of [A.13]. Note that in [A.13] α is a function of U_{LS} as described in appendix B (figure 8) (the dependence of α on U_{GS} is negligible in the range of the severe slugging flow rates).

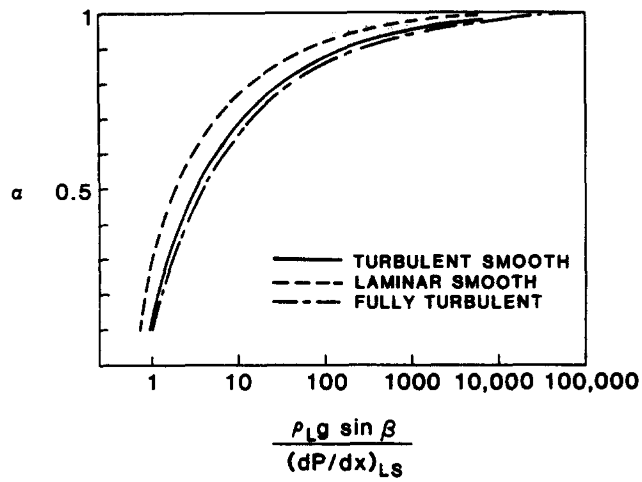

Figure 8. Void fraction in downwards inclined pipe.

As seen the results of [A.13] is fairly close to the experimental boundary to severe slugging I as given by Schmidt *et al.* (1979a) (the broken curve). Boundary b is the transition boundary from stratified flow as calculated using Taitel and Dukler (1976) model. The flow in the inclined pipe must be stratified in order for severe slugging to be possible. The region enclosed by a and below b is the region where severe slugging is possible.

Although severe slugging is possible, in this region, it will not take place if the stability criterion [3] is satisfied. For a given liquid flow rate the pipeline void fraction α can be calculated using figure 8 (appendix B), while α', the void fraction of a Taylor bubble, is fairly constant (see appendix C) and was taken as $\alpha'=0.89$. Equation [3] can now be used to calculated the separator pressure ratio P_s/P_0 above which severe slugging is not possible. In figure 9 three such pressure ratios are plotted: 0.1, 0.5 and 1.0. This indicates that, for example, when the separator pressure ratio is 1 only in the region below the $P_s/P_0=1$ line severe slugging is possible. Interestingly enough, Schmidt data fall right in the range of

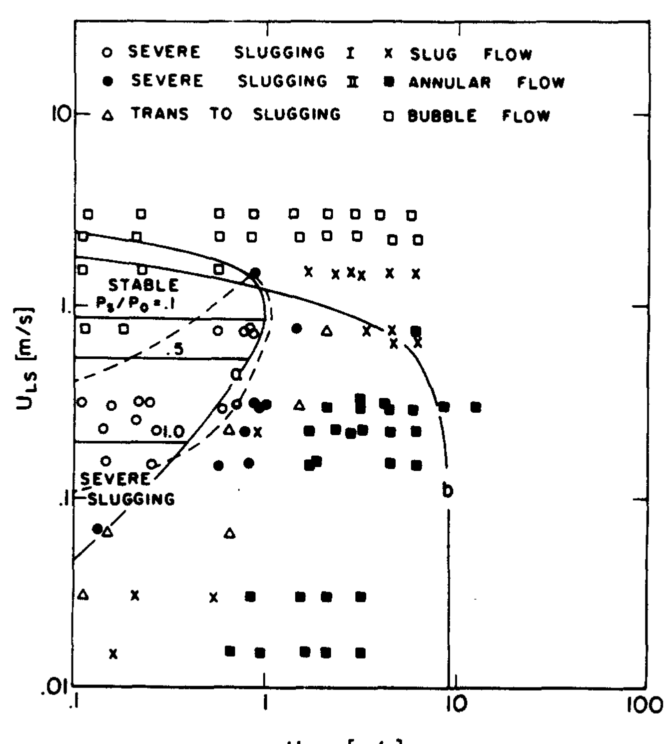

Figure 9. Experimental results, kerosine–air system, 51-mm-diam. pipe, 5° pipeline inclincation, riser 15 m long, pipeline 30 m long (Schmidt 1977; Schmidt *et al.* 1979a).

these pressure ratios, and although the pressure at the top of the riser was not controlled, nor reported by Schmidt (1977), it was in the range of 0.1 to 1. This may explain why in the upper region of the severe slugging zone severe slugging was not observed. Obviously more carefully planned experiments are needed to fully verify this proposed theory.

SUMMARY AND CONCLUSIONS

Severe slugging may occur whenever a pipeline is laid with a downward inclination followed by an upward slope that allows liquid to accumulate at the lower section. This phenomenon is limited to low liquid and gas flow rates at which the flow pattern in the downhill section is stratified. The system analysed here is that of a downsloping pipeline that ends with a vertical riser. For this system severe slugging is possible when the flow pattern in the pipeline is stratified and the liquid flow rate is above the value given by [A.13].

It has been shown that a steady state operation is stable if the separator pressure is high enough to satisfy [4]. If the steady state operation is unstable, a proportional controlled choking valve that follows [10] can stabilize the flow if the proportionality constant is chosen to satisfy [12].

There are situations, however, when the system can operate both under steady state and under unstable severe slugging conditions. Transition from severe slugging to steady state operation can be achieved by temporarily increasing the separator pressure or the proportionality constant K when a choking valve is used.

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APPENDIX A

Simplified severe slugging model

A model for severe slugging was prsented by Schmidt *et al.* (1980). The purpose of such a model is to predict the slug length, slug cycle time, pressure fluctuations, etc.

A somewhat simpler representation of this model is described below with the prime objective to predict the flow rate of liquid above which the tail of the slug reaches the bottom of the riser before the front of the slug reaches the top of the riser. Under such conditions, severe slugging is not possible.

With reference to figure 1, $x(t)$ and $z(t)$ can be calculated using the following.

Hydrostatic pressure

$$P_p = \rho_L g(z - x \sin \beta) + P_s \quad . \qquad [A.1]$$

Volume of the gas in the pipeline

$$V_G = (l - x)\alpha A \quad , \qquad [A.2]$$

where A is the cross-sectional area of the pipeline.

State equation (assuming ideal gas)

$$P_p = \frac{m_G}{V_G} RT \quad , \qquad [A.3]$$

where m_G is the mass of the gas and R is the ideal gas constant.

Conservation of liquid

$$m_L = m_{Li} + \int_0^t U_{LS}\rho_L \, dt \quad . \qquad [A.4]$$

Conservation of gas

$$m_G = m_{Gi} + \int_0^t U_{GSO}\rho_{GO} \, dt \quad , \qquad [A.5]$$

where i refers to the initial condition.

The masses of the liquid and gas at any time can be given in terms of x and z as follows:

$$m_L = \rho_L A(x + z) + (1 - \alpha)\rho_L A(l - x) \quad , \qquad [A.6]$$

$$m_G = \rho_G V_G = \frac{P_s + \rho_L g(z - x \sin \beta)}{RT}(l - x)A\alpha \quad . \qquad [A.7]$$

Note that the initial values of the liquid and gas masses, m_{Gi} and m_{Li}, can also be calculated by [A.6] and [A.7] with $x = x_i$ and $z = z_i$. The determination of these initial values will be discussed later. Note also that the void fraction in the pipe, α, is considered to be known and its calculation is given in appendix B (it is, in principle, the same as suggested by Schmidt *et al.* 1980).

Substituting m_{Gi} from [A.7] into [A.5] and then substituting [A.1], [A.2] and [A.5] into [A.3] gives

$$\left[\frac{P_s}{\rho_L g} + (z - x \sin \beta)\right] (l - x)\alpha = \left[\frac{P_s}{\rho_L g} + (z_i - x_i \sin \beta)\right](l - x_i) + \frac{RT}{\rho_L g}\int_0^t U_{GSO}\rho_{GO} \, dt \quad . \qquad [A.8]$$

Substituting [A.6] for m_L and m_{Li} into [A.4] yields the relation for the liquid conservation

$$z = z_i - \alpha(x - x_i) + \int_0^t U_{LS}\,dt \quad . \qquad [A.9]$$

Substituting [A.9] in [A.8] yields a simple quadratic equation for $x(t)$ as well as $z(t)$.

The prediction of $x(t)$ and $z(t)$ given by [A.8] and [A.9] corresponds to the slug formation step (figure 1). Once the slug reaches the top of the riser ($z = h$) the process is continued as shown in step 2 (figure 2). Thus after $z = h$ the solution for $x(t)$ is obtained directly from [A.8] with $z = h$.

The initial condition for x_i and z_i depends on how much liquid falls back in step 4 (figure 4) which, in turn, depends on the amount of liquid that stays as a film in the blowout step (figure 3). Since the blowout step is similar to a Taylor bubble motion in normal slug flow, the amount of liquid left can be calculated using a slug flow model (see appendix C). As noted previously this is not a straightforward solution since the effective gas and liquid flow rates are not known. However, the void fraction in a Taylor bubble is insensitive to the flow rates. For example, for water and air flowing in 5 cm pipe, $\alpha' = 0.89$ to a very good approximation for all practical flow rates. This means that for a water–air system, about 10% of the liquid falls back. Consider that the fallback is fast. If not, there will be some difference in the solution for a short time period but no difference for longer times once the liquid fallback is completed. Then:
Hydrostatic pressure yields

$$P_p = \rho_L g(z_i - x_i \sin\beta) + P_s \quad . \qquad [A.10]$$

Liquid mass balance requires

$$\alpha x_i + z_i = (1 - \alpha')\,h \quad , \qquad [A.11]$$

while the compression of the gas in the pipeline follows the relation

$$P_p = P_s \frac{l}{l - x_i} \quad . \qquad [A.12]$$

Substituting [A.12] and [A.11] in [A.10] yields a single equation for x_i as well as z_i (using [A.11] again).

Using the calculated values for x_i and z_i it is possible now to solve for x and z as a function of time. If $z = h$ before $x = 0$, then severe slugging will not take place.

We can simplify the procedure by assuming x_i, $z_i = 0$. For a constant mass flow rate of liquid and gas, [A.8] and [A.9] show that one can find conditions for which x is identically zero for all times. Substituting $x = 0$ in [A.8] and [A.5] yields

$$U_{LS} = \frac{\rho_{G0} RT}{\rho_L g \alpha l} U_{GS0} \quad , \qquad [A.13]$$

which shows the liquid flow rate below which severe slugging will not occur.

This is rather a remarkable simple result. Indeed, inclusion of the effect of fallback has only a minor effect on this result.

It is interesting to see that this condition depends on the pipeline length (though not on the riser length). From this point it is somewhat misleading to correlate the boundary of severe slugging on a map with U_{LS}, U_{GS} as coordinates without specifying the pipeline length.

APPENDIX B

Stratified flow model in downward inclination flow

The liquid downflow in the pipeline towards the riser is very closely approximated by a fully developed open channel flow. Schmidt *et al.* (1980) suggested the use of the Manning derivation (Allen 1972). Equally valid is the approach suggested by Taitel & Dukler (1976) for calculating the equilibrium level in stratified flow for the special case where the gas velocity is negligible. In this case a momentum balance of shear stress and gravity on the liquid phase yields

$$\tau_L S_L = \rho_L g A_L \sin\beta \quad , \qquad [B.1]$$

where (as in Taitel & Dukler 1976)

$$\tau_L = f_L \frac{\rho_L U_L^2}{2} \quad . \qquad [B.2]$$

The friction factor f_L can be calculated from the Moody diagram with the appropriate hydraulic diameter. For smooth pipe, for example, the friction factor can be calculated by

$$f_L = C_L \left(\frac{4A_L U_L}{S_L \nu_L}\right)^{-m} \quad , \qquad [B.3]$$

where $C_L = 0.046$, $m=0.2$ for turbulent flow and $C_L = 16$, $m=1$ for laminar flow. A_L, the cross-sectional area of the liquid and S_L, the wetting periphery are given in terms of the equilibrium liquid level h_L:

$$A_L = 0.25D^2[\pi - \cos^{-1}(2\frac{h_L}{D} - 1) + (2\frac{h_L}{D} - 1)\sqrt{1-(2\frac{h_L}{D} - 1)^2} \quad , \qquad [B.4]$$

$$S_L = D\,[\pi - \cos^{-1}(2\frac{h_L}{D} - 1)] \quad . \qquad [B.5]$$

Equation [B.1] can now be solved by trial and error for the equilibrium level h_L. Once h_L is given the void fraction α can be calculated by

$$\alpha = 1 - A_L/A \quad . \qquad [B.6]$$

A general solution can be presented in a dimensionless form

$$\alpha = f\left[\frac{(\rho_L - \rho_G)g \sin\beta}{(\mathrm{d}P/\mathrm{d}x)_{LS}}\right] \quad , \qquad [B.7]$$

where $(\mathrm{d}P/\mathrm{d}x)_{LS}$ is the pressure drop when the liquid flows alone in the pipe, namely

$$\left(\frac{\mathrm{d}P}{\mathrm{d}x}\right)_{LS} = f_{LS}\frac{\rho_L U_{LS}^2}{2} \quad . \qquad [B.8]$$

f_{LS} is the friction factor when the liquid flows alone in the pipe.

The result of [B.7] is plotted in figure 8 for convenience. It includes the smooth pipe case where the friction factor is given by [B.3] and the fully turbulent case where f_L is constant ($f_L/f_{LS}=1$).

APPENDIX C

Slug flow model

Vertical slug flow consists of long Taylor bubbles separated by slugs of liquid. The liquid slugs usually contain small bubbles.

Models for slug flow were presented with various degrees of accuracy by Taitel, Barnea & Dukler (1980), Taitel & Barnea (1983) and recently by Fernandes *et al.* (1983), who proposed a detailed hydrodynamic model for vertical upward slug flow.

In this appendix a simplified model, based primarily on the work of Fernandes *et al.* (1983), is used. The simplification allows a relatively simple solution with little sacrifice of accuracy.

The translational velocity of a Taylor bubble is assumed to be given by (Nicklin *et al.* 1962).

$$U_t = 1.2\ U_S + 0.35\ \sqrt{gD}\quad , \tag{C.1}$$

where U_S is the superficial mixture velocity given by

$$U_S = U_{LS} + U_{GS}\quad . \tag{C.2}$$

A liquid mass balance relative to a coordinate system that moves with the translational velocity U_t yields

$$R_f(U_t + U_f) = R_s(U_t - U_L)\quad , \tag{C.3}$$

where U_L is the liquid velocity in the slug, U_f the film velocity around the Taylor bubble (positive for downward flow), R_s the liquid holdup in the slug, and R_f the liquid holdup in a cross-sectional area of the Taylor bubble and the liquid film.

The void fraction in the liquid is very close to 30% (Barnea & Brauner 1984; Fernandes *et al.* 1983), namely R_S=0.7.

In the liquid slug the relative bubble rise velocity is U_0 (given by [9]). Therefore

$$U_L = U_S - U_0(1 - R_s)\quad . \tag{C.4}$$

The liquid film around the Taylor bubble is considered to be a free falling film for which the film thickness is given by (Wallis 1969)

$$\frac{\delta}{D} = k\ [\frac{\mu_L^2}{D^3 g\,(\rho_L - \rho_G)\rho_L}]^{1/3}\ [\frac{4\Gamma}{\mu_L}]^m\quad , \tag{C.5}$$

where Γ is the mass flow rate per unit peripherial length, $\Gamma=\rho_L U_f \delta$. k and m for laminar flow equal 0.909 and 1/3. For turbulent flow ($Re = 4\Gamma/\mu_L > 1000$) various constants are suggested. Wallis suggested k=0.115, m=0.6. An alternative relation proposed by Belkin (1959) suggests k=0.063, m=2/3. Fernandes *et al.* (1983), on the other hand, strongly recommends the use of the Brotz (1954) relation which suggests k=0.0682, m=2/3 which were the constants used here.

Equation [C.5] can be rearranged in the form

$$U_f = \{\ \frac{(\delta/D)^{1-m}}{k\ [\mu_L^2/D^3 g(\rho_L-\rho_G)\rho_L]^{1/3}\ [4\rho_L D/\mu_L]^m}\}^{1/m}\quad . \tag{C.6}$$

The liquid holdup in the film, R_f, is directly related to the film thickness

$$R_f = 4\frac{\delta}{D} - 4\,(\frac{\delta}{D})^2 \tag{C.7}$$

Equation [C.3] with [C.7], [C.6] and [C.4] can now be solved by trial and error (using standard iteration techniques) to yield the solution for the film velocity U_f and the film liquid holdup R_f.

A continuity balance on the liquid flow rate yields

$$U_{LS} = U_L R_s \frac{l_s}{l_u} - U_f R_f \frac{l_u - l_s}{l_u}\quad , \tag{C.8}$$

whre l_s is the liquid slug length and l_u the slug unit length which results

$$\frac{l_s}{l_u} = -\frac{U_{LS} + U_f R_f}{[U_S - U_0(1-R_s)]\, R_s + U_f R_f} \quad . \qquad \text{[C.9]}$$

Finally, the average liquid holdup in slug flow is

$$\phi = \frac{l_s}{l_u} R_s + (1 - \frac{l_s}{l_u})\, R_f \quad . \qquad \text{[C.10]}$$

The slug model presented here was used in the text for three different purposes:

(1) To calculate the operating line P_S/P_0 as a function of ϕ for given liquid flow rate U_{LS} and gas flow rate, in terms of the atmospheric superficial velocity, U_{GS0}. (Note that $U_{GS} = U_{GS0} P_s / P_0$.) These operating curves are valid for $\phi=0$ to $\phi=0.7$. For $\phi > 0.7$ the flow pattern is that of bubble flow and [8] was used in this region.

(2) To estimate the void fraction α' of the gas bubble that penetrates the riser as a result of the unstable situation. As mentioned for a water–air system, 5 cm pipe, R_f is 0.11 for U_{LS} and U_{GS} variation in the complete range from 0.01 to 10 m/s. This shows that the Taylor bubble void fraction is practically independent of the liquid and gas flow rates and thus a constant value for α' could be used.

(3) To predict the amount of fallback of the liquid after the blowout in severe slugging. As mentioned for a water–air system in 5 cm pipe, $R_f \simeq 0.1$ which yields about 10% of the riser volume as liquid fallback. Schmidt *et al.* (1980) correlated the liquid fallback with superficial gas velocity. An average value of 10% of the riser length is quite close to their experimental results.